

Time:	30 minutes
Marks:	/29 marks
Total Marks:	/43 marks

12 Methods Topic Test 1

4 March 2021

Resource Free

ClassPad calculators are <u>NOT</u> permitted. Formulae Sheet is permitted.

Name:

1. [3, 2 = 5 marks]

a) Sketch the graph of the first derivative of the following function:



b) Here is the graph of a function's second derivative.



How many points of inflection does the original function have? Justify your answer.

2. [2, 3, 3 = 8 marks]

a) If
$$y = x^{\frac{1}{3}} - \frac{2}{x}$$
 find $\frac{dy}{dx}$.

(No need to simplify your answer.)

b)
$$\frac{d}{dx} (4x\sqrt{x^3 - 9})$$
 (No need to simplify your answer.)

c) If
$$g(x) = \frac{2x}{(1-6x^2)^3}$$
 find $g'(0)$.

3. [7 marks]

For the function $y = x^3(4 - x)$:

a) Use calculus to determine the coordinates of any points of inflection on the curve and justify whether it/they are horizontal or not.

b) Describe the behaviour of the function as $x \to \infty$.

4. [2, 6 = 8 marks]

An artist's impression of a new warehouse, based on the architect's plans, is shown in Diagram 1 below. Points A and B are the same height above the base.



The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.

With the coordinate axes as shown below in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.



Diagram 2

a) Given that *AB* is 2*x* metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^3\right)$ square metres.

b) The architect wants to fit the rectangular prism (cuboidal) of maximum volume into the girders. Use calculus to find the value of this maximum volume, in simplest form.



Time:

15 minutes

Marks:

/14 marks

12 Methods Topic Test 1

12 March 2020

Resource Assumed

Formulae Sheet, half an A4 size page of notes and ClassPad calculators <u>are</u> permitted.

Name:

5. [3 marks]

If $y = 2x^3 - 7x$, use differentiation to find the approximate change in y when x changes from 5 to 4.99.

6. [6 marks]

The graph of y = g(x) is shown on the axes below. Identify all points, from those labelled (A to M), which have each of the following properties. Write "none" if no such points exist.



a) g(x) = 0 and g'(x) > 0

- b) g(x) = 0 and g''(x) > 0
- c) g'(x) = 0 and g''(x) = 0
- d) g'(x) = 0 and g''(x) < 0
- e) g'(x) < 0 and g''(x) = 0

7. [2 marks]

Find the maximum and minimum values of the function $f(x) = x^3 + x^2 - 8x - 5$, for $-4 \le x \le 2$.

8. [2, 3 = 5 marks]

A particle starts at the origin and travels in a straight line with velocity v m/s given by $v = (5t - 2)(t + 1), t \ge 0.$

- a) Calculate the initial acceleration of the particle.
- b) Is the particle speeding up or slowing down when t = 0? Justify your answer.